Second Edition

ADVANCED INTERNATIONAL TRADE

Theory and Evidence

ROBERT C. FEENSTRA

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Advanced International Trade

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To Heather and Evan

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ACKNOWLEDGMENTS

A book like this would not be possible without the assistance of many people; indeed, without a career full of teachers and colleagues who have shaped the field and my own views of it. I am fortunate to have had many of these who were all generous with their time and insights. As an undergraduate at the University of British Columbia, I first learned international trade from the original edition of Richard Caves and Ronald Jones, *World Trade and Payments* (Little, Brown, 1973), and then from the "dual" point of view with Alan Woodland. His continuing influence will be clear in this book. I also took several graduate courses in duality theory from Erwin Diewert, and while it took longer for these ideas to affect my research, their impact has been quite profound.

In 1977 I began my graduate work at MIT, where I learned international trade from Jagdish Bhagwati and T. N. Srinivasan, both leaders and collaborators in the field. These courses greatly expanded my knowledge of all aspects of international trade and stimulated me to become a trade economist. The book I have written probably does not do justice to the great breadth of research topics I learned there. These topics are well presented in the textbook by Jagdish Bhagwati, Arvind Panagariya, and T. N. Srinivasan, *Lectures in International Trade* (MIT Press, 1998), to which the reader is referred.

In 1981, I joined Jagdish Bhagwati as a colleague at Columbia University, and for the next five years enjoyed the company of Ronald Findlay, Guillermo Calvo, Robert Mundell, Maury Obstfeld, Stanislaw Wellisz, and other regular attendees at the Wednesday afternoon international seminar. Richard Brecher was also a visitor at Columbia during this period. I am much indebted to these colleagues for the stimulating seminars and conversations. My early work in trade did not stray too far from the familiar two-sector model, but in 1982 I was invited by Robert Baldwin to participate in a conference of the National Bureau of Economic Research (NBER) focusing on the empirical assessment of U.S. trade policies. For this conference I prepared a paper on the voluntary export restraint with Japan in automobiles, and thus my empirically oriented research in trade began.

Following this, I participated in more conferences of the NBER, and since 1992 have directed the International Trade and Investment group there. These conferences have been enormously influential to my research. During the 1980s, there was a large amount of research on trade and trade policy under imperfect competition. Some of this work was done for NBER conferences, and I was able to witness all of it. In the 1990s the focus shifted very substantially to "endogenous" growth models, and this research was again supported by a working group at the NBER and many other researchers. I was fortunate to spend a sabbatical leave in 1989 at the Institute for Advanced Study of the Hebrew University of Jerusalem, along with Wilfred Ethier, Gene Grossman, Paul Krugman, and James Markusen, and under the organization of Elhanan Helpman and Assaf Razin, during which time these growth models were being developed. Furthermore, throughout the 1980s and 1990s there has been an increased awareness of empirical issues in international trade, with fine work by many new colleagues. And during the past decade a great deal of attention has been given to the role of heterogeneous firms in shaping trade patterns and in the gains from trade. So in the span of three decades since beginning my career, I have seen four great waves of research in international trade, and I attempt to summarize all of them in this second edition of the book.

My own research, much of it included here, would not have possible without generous funding from the National Science Foundation, the Ford Foundation, and the Sloan Foundation, as well as the assistance of many graduate students over the years. Some of their research appears in this book, and many others have labored faithfully to construct datasets that have been widely distributed. Let me thank in particular Dorsati Madani, Maria Yang, and Shunli Yao for their past work on datasets, along with David Yue, Roger Butters, Seungjoon Lee, Songhua Lin, and Alyson Ma for their help with the first edition of this textbook. My colleagues and students at UC Davis have been very helpful in discussing this second edition, and I especially thank Vladimir Tyazhelnikov, who provided invaluable research assistance. Finally, I would like to thank a number of other colleagues who contributed to this book: Lee Branstetter provided a number of the empirical exercises, with the STATA programs written by Kaoru Nabeshima; Bin Xu read nearly all the chapters at an early stage and provided detailed comments; and Bruce Blonigen, Donald Davis, Earl Grinols, Doug Irwin, Jiandong Ju, James Levinsohn, Nina Pavcnik, Larry Qiu, James Rauch, and Daniel Trefler provided comments or datasets for specific chapters. I am grateful to these individuals and many others whose input has improved this book.

FOREWORD TO THE SECOND EDITION

This book is intended for a graduate course in international trade. I assume that all readers have completed graduate courses in microeconomics and econometrics. My goal is to bring the reader from that common point up to the most recent research in international trade, in both theory and empirical work. This book is not intended to be difficult, and the mathematics used should be accessible to any graduate student. The material covered will give the reader the skills needed to understand the latest articles and working papers in the field.

The first edition of this textbook was published in 2004, and I am gratified that it has been used in many graduate courses across the United States and around the world. The goals for this second edition are the same as for the first: to provide an accessible treatment of the theory and empirical applications in international trade, thereby bringing the reader up to the frontier of research. But this second edition has another goal, too, and that is to incorporate the most important new material that has appeared in the ten years since the first edition. Foremost among that research are the monopolistic competition model of international trade with heterogeneous firms, due to Melitz (2003), and the Ricardian model of international trade with heterogeneous productivities across countries, due to Eaton and Kortum (2002). While these articles were published prior to the 2004 first edition of this book, they did not appear there other than as references for further reading. That shortcoming is corrected in this second edition.

Because the work of Eaton and Kortum (2002) builds on the Ricardian model with a continuum of goods, we present that model at the end of chapter 3 (drawing upon the presentation in Matsuyama 2008). Then the new chapter 6 has been added, dealing with monopolistic competition and the gravity equation with heterogeneous firms. The presentation in that chapter owes a great deal to articles by Melitz and Redding (2014a) and by Head and Mayer (2014) in the new *Handbook of International Economics, Volume 4.* Melitz and Redding introduce some simplifications to the original Melitz (2003) model and a method of solving it that I rely on in chapter 6, while Head and Mayer show how the gravity equation can be obtained quite generally from a wide range of models, including the Eaton-Kortum model, which is also introduced in chapter 6.

Besides the new chapter 6, other chapters have been rewritten extensively. Chapter 4, dealing with trade in intermediate inputs and wages, now draws on the presentation in my Ohlin Lecture (Feenstra 2010a) and incorporates the work by Grossman and Rossi-Hansberg (2008) on trade in tasks. Chapter 5, focusing on monopolistic competition under homogeneous firms, now includes material on measuring the gains from product variety drawing on Feenstra (1994, 2010b). Chapter 8, which discusses import tariffs and dumping, includes new material on the pass-through of tariffs (or exchange rates) using functional forms that lead to exact expressions and also recent empirical work in this area. Chapter 9, an inquiry into import quotas and export subsidies, naturally lends itself to a fuller treatment of the quality of traded goods, drawing on Feenstra and Romalis (2014). Chapter

10, on the political economy of trade policy, now includes the work of Ossa (2011). Chapter 11, dealing with endogenous growth, includes a brief exposition of growth with heterogeneous firms due to Sampson (2016). And chapter 12, an exploration of multinational corporations and the organization of the firm, has also been rewritten extensively, drawing upon Antràs (2003) and Antràs and Helpman (2004). Much of that material is covered in more depth in the volume by Antràs (2015). Throughout this second edition, I have benefited greatly from researchers providing the surveys and articles that I have incorporated here, for which I offer my thanks.

As before, an instructor's manual that accompanies this book provides solutions to the problems at the end of the chapters.[1] In addition, I have included empirical exercises that replicate the results in some chapters. Completing all of these could be the topic for a second course, but even in a first course there will be a payoff to trying some of the exercises. The data and programs for these can be found on my home page at www.robertfeenstra.info.

The notational conventions from the first edition have been retained in the second edition. I consistently use *subscripts* to refer to goods or factors, whereas *superscripts* refer to consumers or countries. In general, then, *subscripts refer to commodities and superscripts refer to agents*. The index used (h, i, j, k, ℓ, m, or n) will depend on the context. The symbol "c" is used for both costs and consumption, though in some chapters I instead use "$d(p)$" for consumption to avoid confusion. The output of firms is consistently denoted by "y" and exports are denoted by "x", though in some cases "x" denotes inputs. Uppercase letters are used in some cases to denote vectors or matrixes, and in other cases to denote the number of goods (N), factors (M), households (H), or countries (C), and sporadically elsewhere. The symbols α and β are used generically for intercept and slope coefficients, including fixed and marginal labor costs, except that with heterogeneous firms we use the symbol φ for the output produced by one unit of labor and $1/\varphi$ for marginal labor costs.

The contents of several chapters included here have been previously published. Chapter 4 draws on material from my book *Offshoring in the Global Economy: Theory and Evidence* (MIT Press, 2010a) and chapter 5 draws upon the *Scottish Journal of Political Economy* (Feenstra 2002). Some material from chapters 8–10 has appeared in articles published in the *Journal of International Economics* and the *Quarterly Journal of Economics,* and material from chapter 11 has appeared in the *Journal of Development Economics* and in the *American Economic Review.*

[1] Faculty wishing to obtain the instructors manual should contact Princeton University Press.

ADVANCED INTERNATIONAL TRADE

1

Preliminaries: Two-Sector Models

We begin our study of international trade with the classic Ricardian model, which has two goods and one factor (labor). The Ricardian model introduces us to the idea that technological differences across countries matter. In comparison, the Heckscher-Ohlin model dispenses with the notion of technological differences and instead shows how *factor endowments* form the basis for trade. While this may be fine in theory, the model performs very poorly in practice: as we show in the next chapter, the Heckscher-Ohlin model is hopelessly inadequate as an explanation for historical or modern trade patterns unless we allow for technological differences across countries. For this reason, the Ricardian model is as relevant today as it has always been. Our treatment of it in this chapter is a simple review of undergraduate material, but we will present a more sophisticated version of the Ricardian model (with a continuum of goods) in chapter 3.

After reviewing the Ricardian model, we turn to the two-good, two-factor model that occupies most of this chapter and forms the basis of the Heckscher-Ohlin model. We shall suppose that the two goods are traded on international markets, but do not allow for any movements of factors across borders. This reflects the fact that the movement of labor and capital across countries is often subject to controls at the border and is generally much less free than the movement of goods. Our goal in the next chapter will be to determine the pattern of international trade between countries. In this chapter, we simplify things by focusing primarily on *one* country, treating world prices as given, and examine the properties of this two-by-two model. The student who understands all the properties of this model has already come a long way in his or her study of international trade.

RICARDIAN MODEL

Indexing goods by the subscript i, let a_i denote the labor needed per unit of production of each good at home, while a_i^* is the labor need per unit of production in the foreign country, $i = 1, 2$. The total labor force at home is L and abroad is L^*. Labor is perfectly mobile between the industries in each country, but immobile across countries. This means that both goods are produced in the home country only if the wages earned in the two industries are the same. Since the marginal product of labor in each industry is $1/a_i$, and workers are paid the value of their marginal

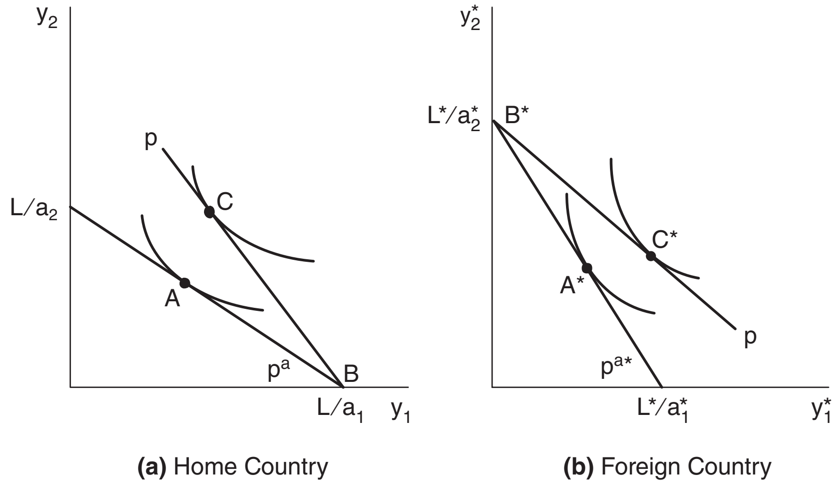

Figure 1.1

products, wages are equalized across industries if and only if $p_1/a_1 = p_2/a_2$, where p_i is the price in each industry. Letting $p = p_1/p_2$ denote the *relative* price of good 1 (using good 2 as the numeraire), this condition is $p = a_1/a_2$.

These results are illustrated in figure 1.1(a) and (b), where we graph the production possibility frontiers (PPFs) for the home and foreign countries. With all labor devoted to good i at home, it can produce L/a_i units, $i = 1, 2$, so this establishes the intercepts of the PPF, and similarly for the foreign country. The slope of the PPF in each country (ignoring the negative sign) is then a_1/a_2 and a_1^*/a_2^*. Under autarky (i.e., no international trade), the equilibrium relative prices p^a and p^{a*} must equal these slopes in order to have both goods produced in both countries, as argued above. Thus, the autarky equilibrium at home and abroad might occur at points A and A^*. Suppose that the home country has a *comparative advantage* in producing good 1, meaning that $a_1/a_2 < a_1^*/a_2^*$. This implies that the home autarky relative price of good 1 is *lower* than that abroad.

Now letting the two countries engage in international trade, what is the equilibrium price p at which world demand equals world supply? To answer this, it is helpful to graph the world relative supply and demand curves, as illustrated in figure 1.2. For the relative price satisfying $p < p^a = a_1/a_2$ and $p < p^{a*} = a_1^*/a_2^*$ both countries are fully specialized in good 2 (since wages earned in that sector are higher), so the world relative supply of good 1 is zero. For $p^a < p < p^{a*}$, the home country is fully specialized in good 1 whereas the foreign country is still specialized in good 2, so that the world relative supply is $(L/a_1)/(L^*/a_2^*)$, as labeled in figure 1.2. Finally, for $p > p^a$ and $p > p^{a*}$, both countries are specialized in good 1. So we see that the world relative supply curve has a “stair-step” shape, which reflects the linearity of the PPFs.

To obtain world relative demand, let us make the simplifying assumption that tastes are identical and homothetic across the countries. Then demand will be independent of the distribution of income *across* the countries. Demand being homothetic means that *relative* demand d_1/d_2 in either country is a downward-sloping function of the relative price p, as illustrated in figure 1.2. In the case we have shown, relative demand intersects relative supply at the world price p that lies *between* p^a and p^{a*}, but this does

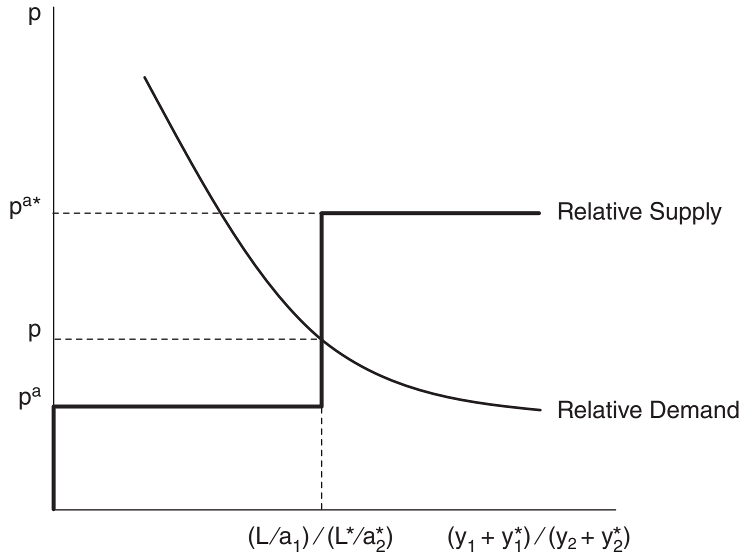

Figure 1.2

not need to occur: instead, we can have relative demand intersect one of the flat segments of relative supply, so that the equilibrium price with trade *equals* the autarky price in one country.[1]

Focusing on the case where $p^a < p < p^{a*}$, we can go back to the PPF of each country and graph the production and consumption points with free trade. Since $p > p^a$, the home country is fully specialized in good 1 at point *B*, as illustrated in figure 1.1(a), and then trades at the relative price p to obtain consumption at point *C*. Conversely, since $p < p^{a*}$, the foreign country is fully specialized in the production of good 2 at point B^* in figure 1.1(b), and then trades at the relative price *p* to obtain consumption at point C^*. Clearly, *both* countries are better off under free trade than they were in autarky: trade has allowed them to obtain a consumption point that is above the PPF.

Notice that the home country exports good 1, which is in keeping with its comparative advantage in the production of that good, $a_1/a_2 < a_1^*/a_2^*$. Thus, *trade patterns are determined by comparative advantage,* which is a deep insight from the Ricardian model. This occurs even if one country has an *absolute disadvantage* in both goods, such as $a_1 > a_1^*$ and $a_2 > a_2^*$, so that more labor is needed per unit of production of *either* good at home than abroad. The reason that it is still possible for the home country to export is that its *wages* will adjust to reflect its productivities: under free trade, its wages are lower than those abroad.[2] Thus, while trade patterns in the Ricardian model are determined by *comparative advantage*, the level of wages across countries is determined by *absolute advantage*.

[1]This occurs if one country is very large. Use figures 1.1 and 1.2 to show that if the home country is very large, then $p = p^a$ and the home country does not gain from trade.

[2]The home country exports good 1, so wages earned with free trade are $w = p/a_1$. Conversely, the foreign country exports good 2 (the numeraire), and so wages earned there are $w^* = 1/a_2^* > p/a_1^*$, where the inequality follow since $p < a_1^*/a_2^*$ in the equilibrium being considered. Then using $a_1 > a_1^*$, we obtain $w = p/a_1 < p/a_1^* < w^*$.

TWO-GOOD, TWO-FACTOR MODEL

While the Ricardian model focuses on technology, the Heckscher-Ohlin model, which we study in the next chapter, focuses on factors of production. So we now assume that there are two factor inputs—labor and capital. Restricting our attention to a single country, we will suppose that it produces two goods with the production functions $y_i = f_i(L_i, K_i)$, $i = 1, 2$, where y_i is the output produced using labor L_i and capital K_i. These production functions are assumed to be increasing, concave, and homogeneous of degree one in the inputs (L_i, K_i).[3] The last assumption means that there are *constant returns to scale* in the production of each good. This will be a maintained assumption for the next several chapters, but we should be point out that it is rather restrictive. It has long been thought that *increasing returns to scale* might be an important reason to have trade between countries: if a firm with increasing returns is able to sell in a foreign market, this expansion of output will bring a reduction in its average costs of production, which is an indication of greater efficiency. Indeed, this was a principal reason why Canada entered into a free-trade agreement with the United States in 1989: to give its firms free access to the large American market. We will return to these interesting issues in chapter 5, but for now, ignore increasing returns to scale.

We will assume that labor and capital are *fully mobile* between the two industries, so we are taking a "long run" point of view. Of course, the amount of factors employed in each industry is constrained by the endowments found in the economy. These resource constraints are stated as

$$\begin{aligned} L_1 + L_2 &\le L, \\ K_1 + K_2 &\le K, \end{aligned} \tag{1.1}$$

where the endowments L and K are fixed. Maximizing the amount of good 2, $y_2 = f_2(L_2, K_2)$, subject to a given amount of good 1, $y_1 = f_1(L_1, K_1)$, and the resource constraints in (1.1) give us $y_2 = h(y_1, L, K)$. The graph of y_2 as a function of y_1 is shown as the PPF in figure 1.3. As drawn, y_2 is a *concave* function of y_1, $\partial^2 h(y_1, L, K)/\partial y_1^2 < 0$. This familiar result follows from the fact that the production functions $f_i(L_i, K_i)$ are assumed to be concave. Another way to express this is to consider all points $S = (y_1, y_2)$ that are feasible to produce given the resource constraints in (1.1). This production possibilities set S is *convex*, meaning that if $y^a = (y_1^a, y_2^a)$and $y^b = (y_1^b, y_2^b)$ are both elements of S, then any point between them $\lambda y^a + (1-\lambda) y^b$ is also in S, for $0 \le \lambda \le 1$.[4]

The production possibilities frontier summarizes the technology of the economy, but in order to determine where the economy produces on the PPF we need to add some assumptions about the market structure. We will assume perfect competition in the product markets and factor markets. Furthermore, we will suppose that product prices are given *exogenously*: we can think of these prices as established on world markets, and outside the control of the "small" country being considered.

[3] Students not familiar with these terms are referred to problems 1.1 and 1.2.

[4] See problems 1.1 and 1.3 to prove the convexity of the production possibilities set, and to establish its slope.

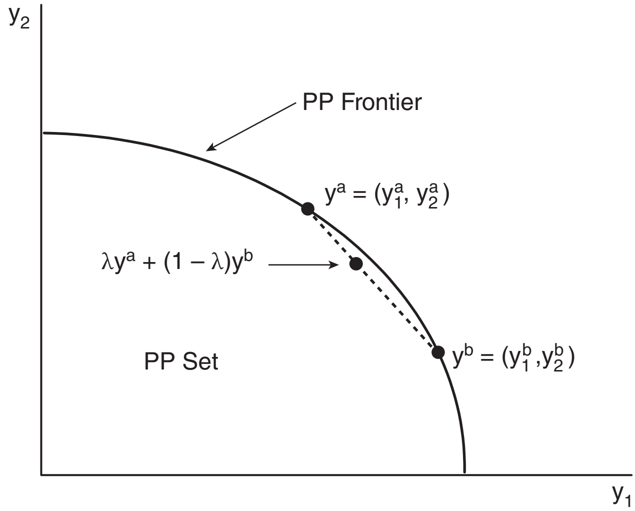

Figure 1.3

GDP FUNCTION

With the assumption of perfect competition, the amounts produced in each industry will maximize gross domestic product (GDP) for the economy: this is Adam Smith's "invisible hand" in action. That is, the industry outputs of the competitive economy will be chosen to maximize GDP:

$$G(p_1,p_2,L,K) = \max_{y_1,y_2} p_1 y_1 + p_2 y_2 \quad \text{s.t.} \quad y_2 = h(y_1,L,K). \tag{1.2}$$

To solve this problem, we can substitute the constraint into the objective function and write it as choosing y_1 to maximize $p_1 y_1 + p_2 h(y_1,L,K)$. The first-order condition for this problem is $p_1 + p_2(\partial h/\partial y_1) = 0$, or,

$$p = \frac{p_1}{p_2} = -\frac{\partial h}{\partial y_1} = -\frac{\partial y_2}{\partial y_1}. \tag{1.3}$$

Thus, the economy will produce where the relative price of good 1, $p = p_1/p_2$, is equal to the slope of the production possibilities frontier.[5] This is illustrated by the point A in figure 1.4, where the line tangent through point A has the slope of (negative) p. An increase in this price will *raise* the slope of this line, leading to a new tangency at point B. As illustrated, then, the economy will produce more of good 1 and less of good 2.

The GDP function introduced in (1.2) has many convenient properties, and we will make use of it throughout this book. To show just one property, suppose that we differentiate the GDP function with respect to the price of good i, obtaining

[5]Notice that the slope of the price line tangent to the PPF (in absolute value) equals the relative price of the good on the *horizontal* axis, or good 1 in figure 1.4.

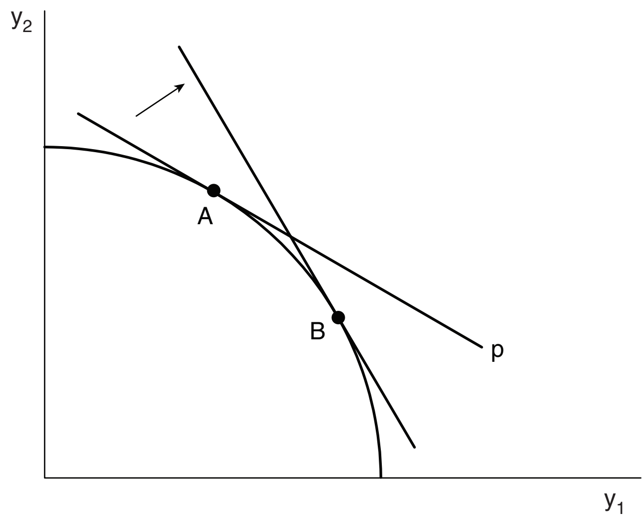

Figure 1.4

$$\frac{\partial G}{\partial p_i} = y_i + \left(p_1 \frac{\partial y_1}{\partial p_i} + p_2 \frac{\partial y_2}{\partial p_i} \right). \quad (1.4)$$

It turns out that the terms in parentheses on the right of (1.4) sum to *zero,* so that $\partial G/\partial p_i = y_i$. In other words, the derivative of the GDP function with respect to *prices* equals the *outputs* of the economy.

The fact that the terms in parentheses sum to zero is an application of the "envelope theorem," which states that when we differentiate a function that has been maximized (such as GDP) with respect to an exogenous variable (such as p_i), then we can *ignore* the changes in the endogenous variables (y_1 and y_2) in this derivative. To prove that these terms sum to zero, totally differentiate $y_2 = h(y_1, L, K)$ with respect to y_1 and y_2 and use (1.3) to obtain $p_1 dy_1 = -p_2 dy_2$, or $p_1 dy_1 + p_2 dy_2 = 0$. This equality must hold for any small movement in y_1 and y_2 around the PPF, and in particular, for the small movement in outputs *induced* by the change in p_i. In other words, $p_1(\partial y_1/\partial p_i) + p_2(\partial y_2/\partial p_i) = 0$, so the terms in parentheses on the right of (1.4) vanish and it follows that $\partial G/\partial p_i = y_i$.[6]

EQUILIBRIUM CONDITIONS

We now want to state succinctly the equilibrium conditions to determine factor prices and outputs. It will be convenient to work with the *unit-cost functions* that are dual to the production functions $f_i(L_i, K_i)$. These are defined by

$$c_i(w, r) = \min_{L_i, K_i \geq 0} \{wL_i + rK_i \,|\, f_i(L_i, K_i) \geq 1\}. \quad (1.5)$$

[6]Other convenient properties of the GDP function are explored in problem 1.4.

In words, $c_i(w,r)$ is the minimum cost to produce one unit of output. Because of our assumption of constant returns to scale, these unit-costs are equal to both marginal costs and average costs. It is easily demonstrated that the unit-cost functions $c_i(w,r)$ are non-decreasing and concave in (w,r). We will write the *solution* to the minimization in (1.5) as $c_i(w,r)=wa_{iL}+ra_{iK}$, where a_{iL} is optimal choice for L_i, and a_{iK} is optimal choice for K_i. It should be stressed that these optimal choices for labor and capital *depend* on the factor prices, so that they should be written in full as $a_{iL}(w,r)$ and $a_{iK}(w,r)$. However, we will usually not make these arguments explicit.

Differentiating the unit-cost function with respect to the wage, we obtain

$$\frac{\partial c_i}{\partial w} = a_{iL} + \left(w\frac{\partial a_{iL}}{\partial w} + r\frac{\partial a_{iK}}{\partial w} \right). \tag{1.6}$$

As we found with differentiating the GDP function, it turns out that the terms in parentheses on the right of (1.6) sum to zero, which is again an application of the "envelope theorem." It follows that the derivative of the unit-costs with respect to the wage equals the labor needed for one unit of production, $\partial c_i/\partial w = a_{iL}$. Similarly, $\partial c_i/\partial r = a_{iK}$.

To prove this result, notice that the constraint in the cost-minimization problem can be written as the isoquant $f_i(a_{iL},a_{iK})=1$. Totally differentiate this to obtain $f_{iL}\,da_{iL} + f_{iK}\,da_{iK} = 0$, where $f_{iL} \equiv \partial f_i/\partial L_i$ and $f_{iK} \equiv \partial f_i/\partial K_i$. This equality must hold for any small movement of labor da_{iL} and capital da_{iK} around the isoquant, and in particular, for the change in labor and capital *induced* by a change in wages. Therefore, $f_{iL}(\partial a_{iL}/\partial w) + f_{iK}(\partial a_{iK}/\partial w) = 0$. Now multiply this through by the product price p_i, noting that $p_i f_{iL} = w$ and $p_i f_{iK} = r$ from the profit-maximization conditions for a competitive firm. Then we see that the terms in parentheses on the right of (1.6) sum to zero.

The first set of equilibrium conditions for the two-by-two economy is that *profits equal zero*. This follows from free entry under perfect competition. The zero-profit conditions are stated as

$$\begin{aligned} p_1 &= c_1(w,r), \\ p_2 &= c_2(w,r). \end{aligned} \tag{1.7}$$

The second set of equilibrium conditions is full employment of both resources. These are the same as the resource constraints (1.1), except that now we express them as equalities. In addition, we will rewrite the labor and capital used in each industry in terms of the derivatives of the unit-cost function. Since $\partial c_i/\partial w = a_{iL}$ is the labor used for *one unit* of production, it follows that the total labor used in $L_i = y_i a_{iL}$, and similarly the total capital used is $K_i = y_i a_{iK}$. Substituting these into (1.1), the full-employment conditions for the economy are written as

$$\begin{aligned} \underbrace{a_{1L}y_1}_{L_1} + \underbrace{a_{2L}y_2}_{L_2} &= L, \\ \underbrace{a_{1K}y_1}_{K_1} + \underbrace{a_{2K}y_2}_{K_2} &= K. \end{aligned} \tag{1.8}$$

Notice that (1.7) and (1.8) together are *four* equations in *four* unknowns, namely, (w,r) and (y_1,y_2). The parameters of these equations, p_1, p_2, L, and K, are given exogenously. Because the unit-cost functions are nonlinear, however, it is not enough to just count equations and unknowns: we need to study these equations in detail to understand whether the solutions are unique and strictly positive, or not. Our task for the rest of this chapter will be to understand the properties of these equations and their solutions.

To guide us in this investigation, there are three key questions that we can ask: (1) what is the solution for factor prices? (2) if prices change, how do factor prices change? (3) if endowments change, how do outputs change? Each of these questions is taken up in the sections that follow. The methods we shall use follow the "dual" approach of Woodland (1977, 1982), Mussa (1979), and Dixit and Norman (1980).

DETERMINATION OF FACTOR PRICES

Notice that our four-equation system above can be decomposed into the zero-profit conditions as *two* equations in *two* unknowns—the wage and rental—and then the full-empoyment conditions, which involve both the factor prices (which affect a_{iL} and a_{iK}) and the outputs. It would be especially convenient if we could uniquely solve for the factor prices from the zero-profit conditions, and then just substitute these into the full-employment conditions. This will be possible when the hypotheses of the following lemma, are satisfied.

LEMMA (FACTOR PRICE INSENSITIVITY)

So long as both goods are produced, and factor intensity reversals (FIRs) do not occur, then each price vector (p_1, p_2) corresponds to unique factor prices (w, r).

This is a remarkable result, because it says that the factor endowments (L, K) do not matter for the determination of (w, r). We can contrast this result with a one-sector economy, with production of $y = f(L, K)$, wages of $w = pf_L$, and diminishing marginal product $f_{LL} < 0$. In this case, any increase in the labor endowments would certainly reduce wages, so that countries with higher labor/capital endowments (L/K) would have lower wages. This is the result we normally expect. In contrast, the above lemma says that in a two-by-two economy, with a fixed product price p, it is possible for the labor force or capital stock to grow *without* affecting their factor prices! Thus, Leamer (1995) refers to this result as "factor price insensitivity." Our goal in this section is to prove the result and also develop the intuition for why it holds.

Two conditions must hold to obtain this result: first, that both goods are produced; and second, that factor intensity reversals (FIRs) do not occur. To understand FIRs, consider figures 1.5 and 1.6. In the first case, presented in figure 1.5, we have graphed the two zero-profit conditions, and the unit-cost lines intersect only *once,* at point A. This illustates the lemma: given (p_1, p_2), there is a *unique* solution for (w, r). But another case is illustrated in figure 1.6, where the unit-cost lines interesect *twice,* at points A and B. Then there are two possible solutions for (w, r), and the result stated in the lemma no longer holds.

The case where the unit-cost lines intersect more than once corresponds to "factor intensity reversals." To see where this name comes from, let us compute the labor and capital requirements in the two industries. We have already shown that a_{iL} and a_{iK} are the derivatives of the unit-cost function with respect to factor prices, so it follows that the vectors (a_{iL}, a_{iK}) are the *gradient vectors* to the iso-cost curves for the two industries

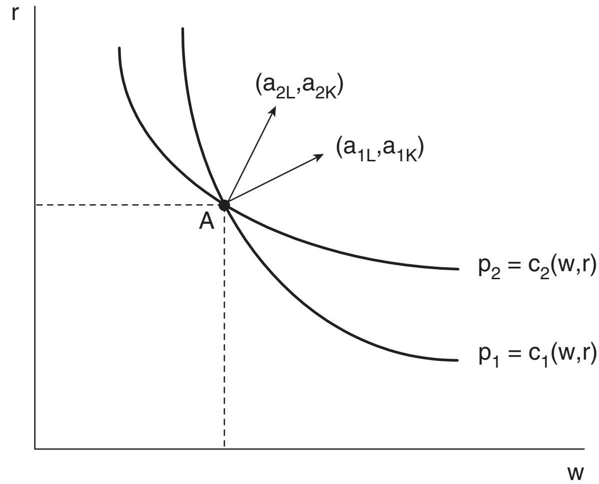

Figure 1.5

in figure 1.5. Recall from calculus that gradient vectors point in the direction of the maximum increase of the function in question. This means that they are *orthogonal* to their respective iso-cost curves, as shown by (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) at point *A*. Each of these vectors has slope (a_{iK}/a_{iL}), or the capital-labor ratio. It is clear from figure 1.5 that (a_{1L}, a_{1K}) has a smaller slope than (a_{2L}, a_{2K}), which means that *industry 2 is capital intensive*, or equivalently, *industry 1 is labor intensive.*[7]

In figure 1.6, however, the situation is more complicated. Now there are two sets of gradient vectors, which we label by (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) at point *A* and by (b_{1L}, b_{1K}) and (b_{2L}, b_{2K}) at point *B*. A close inspection of the figure will reveal that industry 1 is *labor intensive* $(a_{1K}/a_{1L} < a_{2K}/a_{2L})$ at point *A*, but is *capital intensive* $(b_{1K}/b_{1L} > b_{2K}/b_{2L})$ at point *B*. This illustrates a *factor intensity reversal*, whereby the comparison of factor intensities changes at different factor prices.

While FIRs might seem like a theoretical curiosum, they are actually quite realistic. Consider the footwear industry, for example. While much of the footwear in the world is produced in developing nations, the United States retains a small number of plants. For sneakers, New Balance has a plant in Norridgewock, Maine, where employers earn about $14 per hour.[8] Some operate computerized equipment with up to twenty sewing machine heads running at once, while others operate automated stitchers guided by cameras, which allow one person to do the work of six. This is a far cry from the plants in Asia that produce shoes for Nike, Reebok, and other U.S. producers, using century-old technology and paying less than $1 per hour. The technology used to make sneakers in Asia is like that of industry 1 at point *A* in figure 1.5, using labor-intensive

[7] Alternatively, we can totally differentiate the zero-profit conditions, holding prices fixed, to obtain $0 = a_{iL}dw + a_{iK}dr$. It follows that the slope of the iso-cost curve equals $dr/dw = -a_{iL}/a_{iK} = -L_i/K_i$. Thus, the slope of each iso-cost curve equals the relative demand for the factor on the horizontal axis, whereas the slope of the gradient vector (which is orthogonal to the iso-cost curve) equals the relative demand for the factor on the vertical axis.

[8] The material that follows is drawn from Aaron Bernstein, "Low-Skilled Jobs: Do They Have to Move?" *Business Week,* February 26, 2001, pp. 94–95.

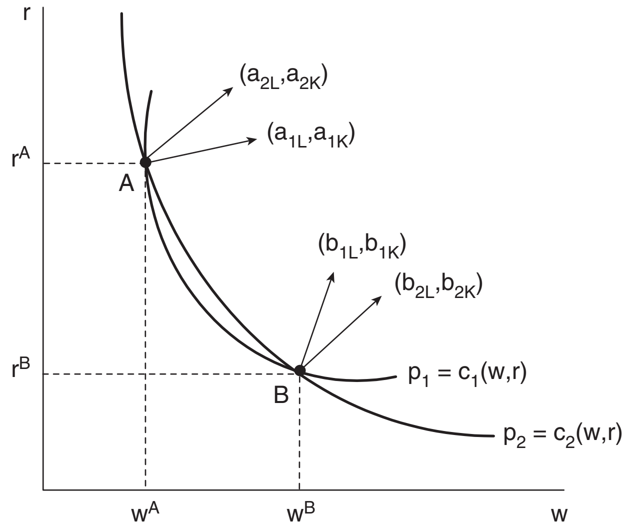

Figure 1.6

technology and paying low wages w^A, while industry 1 in the United States is at point *B*, paying higher wages w^B and using a capital-intensive technology.

As suggested by this discussion, when there are two possible solutions for the factor prices such as points *A* and *B* in figure 1.6, then some countries can be at one equilibrium and others countries at the other. How do we know which country is where? This is a question that we will answer at the end of the chapter, where we will argue that a *labor-abundant* country will likely be at equilibrium *A* of figure 1.6, with a low wage and high rental on capital, whereas a *capital-abundant* country will be at equilibrium *B*, with a high wage and low rental. Generally, to determine the factor prices in each country we will need to examine its full-employment conditions in addition to the zero-profit conditions.

Let us conclude this section by returning to the simple case of no FIR, in which the lemma stated above applies. What are the implications of this result for the determination of factor prices under free trade? To answer this question, let us sketch out some of the assumptions of the Heckscher-Ohlin model, which we will study in more detail in the next chapter. We assume that there are two countries, with identical technologies but different factor endowments. We continue to assume that labor and capital are the two factors of production, so that under free trade the equilibrium conditions (1.7) and (1.8) apply in *each* country with the *same* product prices (p_1, p_2). We can draw figure 1.5 for each country, and in the absence of FIR, this *uniquely* determines the factor prices in each countries. In other words, the wage and rental determined by figure 1.5 are *identical* across the two countries. We have therefore proved the factor price equalization (FPE) theorem, which is stated as follows.

FACTOR PRICE EQUALIZATION THEOREM (SAMUELSON 1949)

Suppose that two countries are engaged in free trade, having identical technologies but different factor endowments. If both countries produce both goods and FIRs do not occur, then the factor prices (w, r) are equalized across the countries.

The FPE theorem is a remarkable result because it says that *trade in goods* has the ability to equalize factor prices: in this sense, trade in goods is a "perfect substitute" for trade in factors. We can again contrast this result with that obtained from a one-sector economy in both countries. In that case, equalization of the product price through trade would certainly not equalize factor prices: the labor-abundant country would be paying a lower wage. Why does this outcome *not occur* when there are two sectors? The answer is that the labor-abundant country can *produce more of, and export,* the labor-intensive good. In that way it can fully employ its labor while still paying the same wages as a capital-abundant country. In the two-by-two model, the opportunity to disproportionately produce more of one good than the other, while exporting the amounts not consumed at home, is what allows factor price equalization to occur. This intuition will become even clearer as we continue to study the Heckscher-Ohlin model in the next chapter.

CHANGE IN PRODUCT PRICES

Let us move on now to the second of our key questions of the two-by-two model: if the product prices change, how will the factor prices change? To answer this, we perform comparative statics on the zero-profit conditions (1.7). Totally differentiating these conditions, we obtain

$$dp_i = a_{iL}dw + a_{iK}dr \Rightarrow \frac{dp_i}{p_i} = \frac{wa_{iL}}{c_i}\frac{dw}{w} + \frac{ra_{iK}}{c_i}\frac{dr}{r}, i = 1,2. \tag{1.9}$$

The second equation is obtained by multiplying and dividing like terms, and noting that $p_i = c_i(w,r)$. The advantage of this approach is that it allows us to express the variables in terms of *percentage changes,* such as $d\ln w = dw/w$, as well as *cost-shares.* Specifically, let $\theta_{iL} = wa_{iL}/c_i$ denote the cost-share of labor in industry *i*, while $\theta_{iK} = ra_{iK}/c_i$ denotes the cost-share of capital. The fact that costs equal $c_i = wa_{iL} + ra_{iK}$ ensures that the shares sum to unity, $\theta_{iL} + \theta_{iK} = 1$. In addition, denote the percentage changes by $dw/w = \hat{w}$ and $dr/r = \hat{r}$. Then (1.9) can be re-written as

$$\hat{p}_i = \theta_{iL}\hat{w} + \theta_{iK}\hat{r}, i = 1,2. \tag{1.9'}$$

Expressing the equations using these cost-shares and percentage changes follows Jones (1965) and is referred to as the "Jones algebra." This system of equations can be written in matrix form and solved as

$$\begin{pmatrix}\hat{p}_1\\ \hat{p}_2\end{pmatrix} = \begin{pmatrix}\theta_{1L} & \theta_{1K}\\ \theta_{2L} & \theta_{2K}\end{pmatrix}\begin{pmatrix}\hat{w}\\ \hat{r}\end{pmatrix} \Rightarrow \begin{pmatrix}\hat{w}\\ \hat{r}\end{pmatrix} = \frac{1}{|\theta|}\begin{pmatrix}\theta_{2K} & -\theta_{1K}\\ -\theta_{2L} & \theta_{1L}\end{pmatrix}\begin{pmatrix}\hat{p}_1\\ \hat{p}_2\end{pmatrix}, \tag{1.10}$$

where $|\theta|$ denotes the determinant of the two-by-two matrix on the left. This determinant can be expressed as

$$\begin{aligned}|\theta| &= \theta_{1L}\theta_{2K} - \theta_{1K}\theta_{2L}\\ &= \theta_{1L}(1-\theta_{2L}) - (1-\theta_{1L})\theta_{2L}\\ &= \theta_{1L} - \theta_{2L} = \theta_{2K} - \theta_{1K}\end{aligned} \tag{1.11}$$

where we have repeatedly made use of the fact that $\theta_{iL} + \theta_{iK} = 1$.

In order to fix ideas, let us assume henceforth that *industry 1 is labor intensive.* This implies that its labor cost-share in industry 1 exceeds that in industry 2, $\theta_{1L} - \theta_{2L} > 0$, so that $|\theta| > 0$ in (1.11).[9] Furthermore, suppose that the relative price of good 1 *increases,* so that $\hat{p} = \hat{p}_1 - \hat{p}_2 > 0$. Then we can solve for the change in factor prices from (1.10) and (1.11) as

$$\hat{w} = \frac{\theta_{2K}\hat{p}_1 - \theta_{1K}\hat{p}_2}{|\theta|} = \frac{(\theta_{2K} - \theta_{1K})\hat{p}_1 + \theta_{1K}(\hat{p}_1 - \hat{p}_2)}{(\theta_{2K} - \theta_{1K})} > \hat{p}_1, \tag{1.12a}$$

since $\hat{p}_1 - \hat{p}_2 > 0$, and,

$$\hat{r} = \frac{\theta_{1L}\hat{p}_2 - \theta_{2L}\hat{p}_1}{|\theta|} = \frac{(\theta_{1L} - \theta_{2L})\hat{p}_2 - \theta_{2L}(\hat{p}_1 - \hat{p}_2)}{(\theta_{1L} - \theta_{2L})} < \hat{p}_2, \tag{1.12b}$$

since $\hat{p}_1 - \hat{p}_2 > 0$.

From the result in (1.12a), we see that the wage increases *by more* than the price of good 1, $\hat{w} > \hat{p}_1 > \hat{p}_2$. This means that workers can afford to buy more of good 1 (w/p_1 has gone up), as well as more of good 2 (w/p_2 has gone up). When labor can buy more of *both goods* in this fashion, we say that the *real wage* has increased. Looking at the rental on capital in (1.12b), we see that the rental r changes by *less than* the price of good 2. It follows that capital-owner can afford less of good 2 (r/p_2 has gone down), and also less of good 1 (r/p_1 has gone down). Thus the *real return to capital* has fallen. We can summarize these results with the following theorem.

STOLPER-SAMUELSON (1941) THEOREM

An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduce the real return to the other factor.

To develop the intuition for this result, let us go back to the differentiated zero-profit conditions in (1.9′). Since the cost-shares add up to unity in each industry, we see from equation (1.9′) that $\hat{p}_i$ is a *weighted average* of the factor price changes $\hat{w}$ and $\hat{r}$. This implies that $\hat{p}_i$ necessarily lies in between $\hat{w}$ and $\hat{r}$. Putting these together with our assumption that $\hat{p}_1 - \hat{p}_2 > 0$, it is therefore clear that

$$\hat{w} > \hat{p}_1 > \hat{p}_2 > \hat{r}. \tag{1.13}$$

Jones (1965) has called this set of inequalities the "magnification effect": they show that any change in the product price has a *magnified effect* on the factor prices. This is an extremely important result. Whether we think of the product price change as due to export opportunities for a country (the export price goes up), or due to lowering import tariffs (so the import price goes down), the magnification effect says that there will be both gainers and losers due to this change. Even though we will argue in chapter 6 that there are gains from trade in some overall sense, it is still the case that trade opportunities have strong *distributional* consequences, making some people worse off and some better off!

[9] As an exercise, show that $L_1/K_1 > L_2/K_2 \Leftrightarrow \theta_{1L} > \theta_{2L}$. This is done by multipying the numerator and denominator on both sides of the first inequality by like terms, so as to convert it into cost-shares.

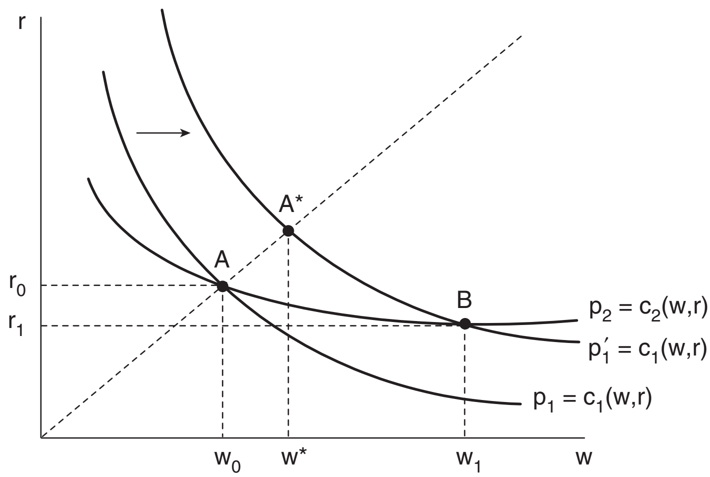

Figure 1.7

We conclude this section by illustrating the Stolper-Samuelson theorem in figure 1.7. We begin with an initial factor price equilibrium given by point A, where industry 1 is labor intensive. An increase in the price of that industry will shift out the iso-cost curve, and as illustrated, move the equilibrium to point B. It is clear that the wage has gone up, from w_0 to w_1, and the rental has declined, from r_0 to r_1. Can we be sure that the wage has increased in percentage terms *by more* than the relative price of good 1? The answer is yes, as can be seen by drawing a ray from the origin through the point A. Because the unit-cost functions are homogeneous of degree one in factor prices, moving along this ray increases p and (w, r) in the same proportion. Thus, at the point A^*, the increase in the wage exactly matched the percentage change in the price p_1. But it is clear that the equilibrium wage increases by *more*, $w_1 > w^*$, so the percentage increase in the wage *exceeds* that of the product price, which is the Stolper-Samuelson result.

CHANGES IN ENDOWMENTS

We turn now to the third key question: if endowments change, how do the industry outputs change? To answer this, we hold the product prices *fixed* and totally differentiate the full-employment conditions (1.8) to obtain

$$\begin{aligned} a_{1L}dy_1 + a_{2L}dy_2 &= dL, \\ a_{1K}dy_1 + a_{2K}dy_2 &= dK. \end{aligned} \tag{1.14}$$

Notice that the a_{ij} coefficients *do not* change, despite the fact that they are functions of the factor prices (w, r). These coefficients are fixed because p_1 and p_2 do not change, so from our earlier lemma, the factor prices are also fixed.

By rewriting the equations in (1.14) using the "Jones algebra," we obtain

$$\begin{aligned} \frac{y_1 a_{1L}}{L}\frac{dy_1}{y_1} + \frac{y_2 a_{2L}}{L}\frac{dy_2}{y_2} &= \frac{dL}{L} \\ \frac{y_1 a_{1K}}{K}\frac{dy_1}{y_1} + \frac{y_2 a_{2K}}{K}\frac{dy_2}{y_2} &= \frac{dK}{K} \end{aligned} \quad \Rightarrow \quad \begin{aligned} \lambda_{1L}\hat{y}_1 + \lambda_{2L}\hat{y}_2 &= \hat{L} \\ \lambda_{1K}\hat{y}_1 + \lambda_{2K}\hat{y}_2 &= \hat{K}. \end{aligned} \tag{1.14′}$$

To move from the first set of equations to the second, we denote the percentage changes $dy_1/y_1 = \hat{y}_1$, and likewise for all the other variables. In addition, we define $\lambda_{iL} \equiv (y_i a_{iL}/L) = (L_i/L)$, which measures the *fraction of the labor force employed in industry i,* where $\lambda_{1L} + \lambda_{2L} = 1$. We define λ_{iK} analogously as the fraction of the capital stock employed in industry *i*.

This system of equations is written in matrix form and solved as

$$\begin{bmatrix} \lambda_{1L} & \lambda_{2L} \\ \lambda_{1K} & \lambda_{2K} \end{bmatrix} \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} = \begin{pmatrix} \hat{L} \\ \hat{K} \end{pmatrix} \Rightarrow \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} = \frac{1}{|\lambda|} \begin{bmatrix} \lambda_{2K} & -\lambda_{2L} \\ -\lambda_{1K} & \lambda_{1L} \end{bmatrix} \begin{pmatrix} \hat{L} \\ \hat{K} \end{pmatrix}, \tag{1.15}$$

where $|\lambda|$ denotes the determinant of the two-by-two matrix on the left, which is simplified as

$$\begin{aligned} |\lambda| &= \lambda_{1L}\lambda_{2K} - \lambda_{2L}\lambda_{1K} \\ &= \lambda_{1L}(1-\lambda_{1K}) - (1-\lambda_{1L})\lambda_{1K} \\ &= \lambda_{1L} - \lambda_{1K} = \lambda_{2K} - \lambda_{2L}, \end{aligned} \tag{1.16}$$

where we have repeatedly made use of the fact that $\lambda_{1L} + \lambda_{2L} = 1$ and $\lambda_{1K} + \lambda_{2K} = 1$.

Recall that we assumed *industry 1 to be labor intensive.* This implies that the share of the labor force employed in industry 1 exceeds the share of the capital stock used there, $\lambda_{1L} - \lambda_{1K} > 0$, so that $|\lambda| > 0$ in (1.16).[10] Suppose further that the endowments of labor is increasing, while the endowments of capital remains fixed such that $\hat{L} > 0$, and $\hat{K} = 0$. Then we can solve for the change in outputs from (1.15)–(1.16) as

$$\hat{y}_1 = \frac{\lambda_{2K}}{(\lambda_{2K} - \lambda_{2L})}\hat{L} > \hat{L} > 0 \quad \text{and} \quad \hat{y}_2 = \frac{-\lambda_{1K}}{|\lambda|}\hat{L} < 0. \tag{1.17}$$

From (1.17), we see that the output of the labor-intensive industry 1 expands, whereas the output of industry 2 contracts. We have therefore established the Rybczynski theorem.

RYBCZYNSKI (1955) THEOREM

An increase in a factor endowment will increase the output of the industry using it intensively, and decrease the output of the other industry.

To develop the intuition for this result, let us write the full-employment conditions in vector notation as:

$$\begin{pmatrix} a_{1L} \\ a_{1K} \end{pmatrix} y_1 + \begin{pmatrix} a_{2L} \\ a_{2K} \end{pmatrix} y_2 = \begin{pmatrix} L \\ K \end{pmatrix}. \tag{1.8′}$$

We have already illustrated the gradient vectors (a_{iL}, a_{iK}) to the iso-cost curves in figure 1.5 (with not FIR). Now let us take these vectors and regraph them, in figure

[10] As an exercise, show that $L_1/K_1 > L/K > L_2/K_2 \Leftrightarrow \lambda_{1L} > \lambda_{1K}$ and $\lambda_{2K} > \lambda_{2L}$.

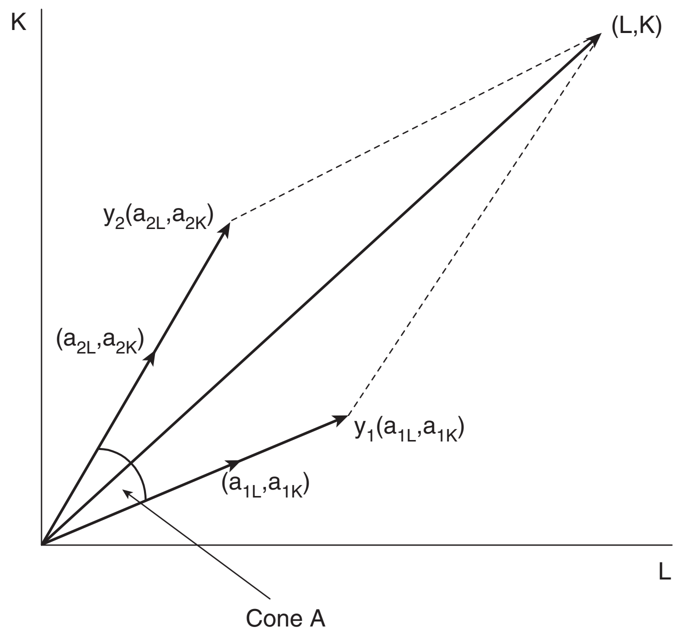

Figure 1.8

1.8. Multiplying each of these by the output of their respective industries, we obtain the total labor and capital demands $y_1(a_{1L}, a_{1K})$ and $y_2(a_{2L}, a_{2K})$. Summing these as in (1.8′) we obtain the labor and capital endowments (L, K). But this exercise can also be performed in reverse: for any endowment vector (L, K), there will be a *unique* value for the outputs (y_1, y_2) such that when (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) are multiplied by these amounts, they will sum to the endowments.

How can we be sure that the outputs obtained from (1.8′) are positive? It is clear from figure 1.8 that the outputs in both industries will be positive if and only if the endowment vector (L, K) lies *in between* the factor requirement vectors (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}). For this reason, the space spanned by these two vectors is called a "cone of diversification," which we label by cone A in figure 1.8. In contrast, if the endowment vector (L, K) lies *outside* of this cone, then it is *impossible* to add together any positive multiples of the vectors (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) and arrive at the endowment vector. So if (L, K) lies outside of the cone of diversification, then it must be that only *one good* is produced. At the end of the chapter, we will show how to determine which good it is.[11] For now, we should just recognize that when only one good is produced, the factor prices are determined by the marginal products of labor and capital as in the one-sector model, and will certainly depend on the factor endowments.

Now suppose that the labor endowment increases to $L' > L$, with no change in the capital endowment, as shown in figure 1.9. Starting from the endowments (L', K), the *only* way to add up multiples of (a_{1L}, a_{1K}) and (a_{2L}, a_{2K}) and obtain the endowments is to *reduce* the output of industry 2 to y_2', and *increase* the output of industry 1 to y_1'. This means that not only does industry 1 absorb the entire amount of the extra labor

[11] See problem 1.5.